Chapter 1: The Heart of the Pulsar

Nobita appeared on the surface of a pulsar--a star which pulsed light from the midst of spinning dramatically in rhythmic bursts. The surface had an intense gravitational pull plus the relentless spin. The intense gravitational pull and the relentless spin left Nobita awestruck. "How does this thing not collapse?" he asked.

Doraemon smiled. "A pulsar is a delicate balance of quantum degeneracy pressure and gravitational force. It's a cosmic tug-of-war that only quantum physics can explain."

Q1:Derive the Tolman-Oppenheimer-Volkoff (TOV) equation and explain how it depicts the equilibrium in a pulsar.

The Tolman-Oppenheimer-Volkoff (TOV) equation describes the balance between the gravitational force and pressure in a spherically symmetric body of matter, such as a neutron star. The equation is derived by solving the Einstein Equations for a general time-invariant, spherically symmetric metric , using Einstein's Field Equations together with the condition of energy - momentum conservation one obtains the TOV equation.

THE TOV EQUATION

https://en.wikipedia.org/wiki/Tolman%E2%80%93Oppenheimer%E2%80%93Volkoff_equation

https://www.as.utexas.edu/astronomy/education/spring13/bromm/secure/TOC_Supplement.pdf

Derivation from General Relativity

Let us assume a static, spherically symmetric perfect fluid.

$$c^2 d\tau^2 = g_{\mu\nu}\, dx^\mu dx^\nu = e^\nu c^2 dt^2 - e^\lambda dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2$$

$$T_0^0 = \rho c^2$$

$$T_i^i = -P\delta_i^i$$

$\rho(r)$ = fluid density and $P(r)$ = fluid pressure

By solving,

$$\frac{8\pi G}{c^4}\rho c^2 e^\nu = \frac{e^\nu}{r^2}\left(1 - \frac{d}{dr}[r e^{-\lambda}]\right)$$

By integrating from 0 to r, we obtain

$$e^{-\lambda} = 1 - \frac{2Gm}{rc^2}$$

We can simplify further,

$$\frac{d\nu}{dr} = \frac{1}{r}\left(1 - \frac{2Gm}{c^2 r}\right)^{-1}\left(\frac{2Gm}{c^2 r} + \frac{8\pi G}{c^4} r^2 P\right)$$

$$\frac{dP}{dr} = \frac{-1}{r}\left(\frac{\rho c^2 + P}{2}\right)\left(\frac{2Gm}{c^2 r} + \frac{8\pi G}{c^4} r^2 P\right)\left(1 - \frac{2Gm}{c^2 r}\right)^{-1}$$

Rearranging we get the final equation,

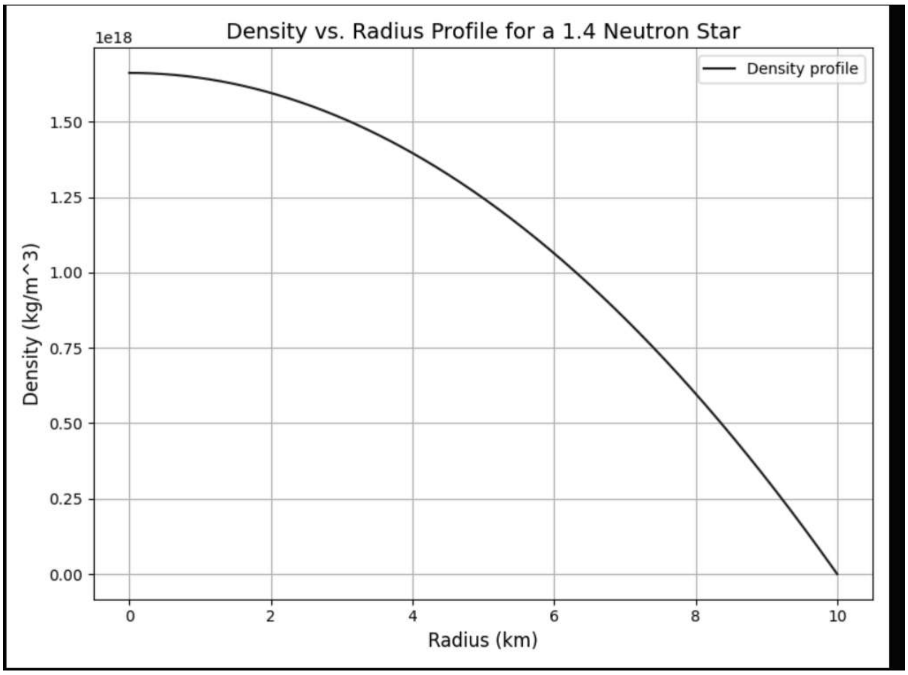
$$\boxed{\frac{dP}{dr} = -\frac{G}{r^2}\left(\rho + \frac{P}{c^2}\right)\left(m + \frac{4\pi r^3 P}{c^2}\right)\left(1 - \frac{2Gm}{c^2 r}\right)^{-1}}$$

$$\frac{dP}{dr} = -\frac{G\epsilon(r)m(r)}{c^2 r^2}\left[1 + \frac{P(r)}{\epsilon(r)}\right]\left[1 + \frac{4\pi r^3 P(r)}{m(r)c^2}\right]\left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1}.$$

This equation summarizes the balance between the inward pull of gravity and outward push of pressure. In a pulsar , the densities are extreme, these relativistic effects are necessary to determine why the star doesn't collapse.

2. Plot the density vs. radius profile for a neutron star of 1.4 solar masses

Python Program Link-1



3. If dark matter particles interacted with the core, how would they modify the equation of state?

Dark Matter particles , for example Weakly interacting Massive Particles (WIMPS) add pressure and density , if weakly interacting they soften the EOS (Equation os state) , reducing maximum neutron star mass. It would introduce pressure and energy density components , changing the objects structure , mass and radius, depending on the strength and nature of the interaction between dark matter and normal matter particles within the core.

Modifications of the Equation of state:

https://journals.aps.org/prd/abstract/10.1103/PhysRevD.97.123007#:~:text=Abstract,are%20explored%20in%20an%20Appendix.

Chapter 2: A Whisper from the Dark

The Time Signal Detector on Doraemon's wrist beeped erratically. "The pulsar's signal timing is off," he said. "Dark matter must be at work."

"What's dark matter, again?" asked Nobita, his curiosity piqued.

"It's a mysterious, invisible form of matter that interacts gravitationally but not electromagnetically. Pulsars are sensitive enough to detect its subtle quantum effects."

Q1: Explain how pulsars can be considered as cosmic clocks, deriving equations for their spin down rate?

Cosmic clocks are often used to describe pulsars, which emit regular pulses of radiation at extraordinarily precise intervals as a result of rapid rotation as neutron stars, enabling astronomers to measure time with great accuracy through the detection of the incoming times of these pulses; in this regard, the extreme stability exists more in the case of millisecond pulsars, which makes them useful in areas such as the detection of gravitational waves and measuring cosmic distances.

One of the key reasons pulsars are considered precise cosmic clocks is their **extraordinary rotational stability.** The spin-down rate of most pulsars is extremely gradual and predictable, allowing scientists to measure time intervals with remarkable accuracy. Millisecond pulsars, which rotate hundreds of times per second, are especially stable, rivaling atomic clocks in precision.

This stability enables various astrophysical and scientific applications. Pulsars help test general relativity, detect gravitational waves, and improve navigation systems. In pulsar timing arrays, multiple pulsars are monitored to detect ripples in spacetime caused by massive cosmic events. Additionally, pulsar-based timekeeping has been proposed for deep-space navigation.

Thus, the predictable and precise nature of pulsar rotations makes them invaluable as cosmic timekeepers, providing insight into fundamental physics and the nature of spacetime itself.

For the derivation, let us first define a few terms and their formulas:

1. Moment of Inertia (I) and Rotational Kinetic Energy (E_{rot}): Quantity expressed by the body resisting angular acceleration is called the MOI of the body. Rotational kinetic energy is the energy contained in a body by the virtue of its motion, in this case rotational motion. The relationship between them is simple:

$$E_{rot} = \frac{1}{2} I \Omega^2$$

Here, Ω is the angular velocity.

2. Radiated power a pulsar (P): It is the amount of the electromagnetic energy radiated by a pulsar per unit time. The relation between power and angular velocity(Ω) is given by:

$$P = \frac{2}{3} \frac{\mu^2 \Omega^4}{c^3}$$

Here μ is the magnetic dipole moment given by the formula BR^3 where B is the magnetic field intensity and R is the radius.

3. $d(E_{rot})/dt$: Rate of change of Rotational Kinetic Energy is essentially the additive inverse of the power radiated by the pulsar (=-P).

The derivation starts with substituting terms into the last equation, being $d(E_{rot})/dt=-P$. It brings us to the equation:

$$\frac{d\left[\frac{I\Omega^2}{2}\right]}{dt} = -\frac{2}{3} \frac{\mu^2 \Omega^4}{c^3}$$

On the LHs let us take IΩ common, which leaves us with:

$$I\Omega \cdot \frac{d\Omega}{dt} = -\frac{2}{3}\,\mu^2\frac{\Omega^4}{c^3}$$

On solving further:

$$\dot{\Omega} = \frac{d\Omega}{dt} = -\frac{2}{3}\,\mu^2\frac{\Omega^3}{Ic^3}$$

$$P = \frac{2\pi}{\Omega}, \quad \text{differentiating both sides}$$

$$\dot{P} = \frac{2\pi}{\Omega^2}\cdot\dot{\Omega}$$

$$\dot{P} = \frac{4\pi}{3}\cdot\frac{\mu^2\Omega}{Ic^3}$$

$$\dot{P} = \frac{4\pi}{3}\cdot\frac{\mu^2}{Ic^3}\cdot\frac{2\pi}{P}$$

$$\dot{P}P = \frac{8\pi^2\mu^2}{3Ic^3}$$

(*symbols with a dot on top represent their first derivative)*

Finally the characteristic age of a pulsar is given by the equation:

$$\tau = \frac{P}{2\dot{P}} = \frac{P}{2dP/dt}$$

But this operated under 3 underlying assumptions:

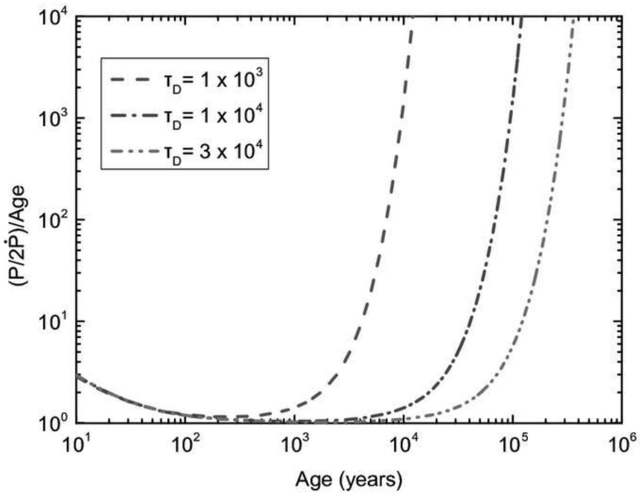
- The pulsar initial spin period was much smaller than the one observed today
- There is no magnetic field decay
- The magnetic braking can be approximated by the energy loss a spinning dipolar magnet would experience in a perfect vacuum (in this case, the braking index, *n* = 3.)

The “correct” formula can be:

$$\tau = \frac{P}{(n-1)\dot{P}} - \frac{P_0}{(n-1)\dot{P}_0}$$

In 2007 the Crab pulsar had a period of 0.0331 sec and a period derivative of 4.22×10^{-13}s/s. The characteristic age is around 1240 years. The supernova that produced the pulsar was in 1054 AD, yielding an age of ~950 years.

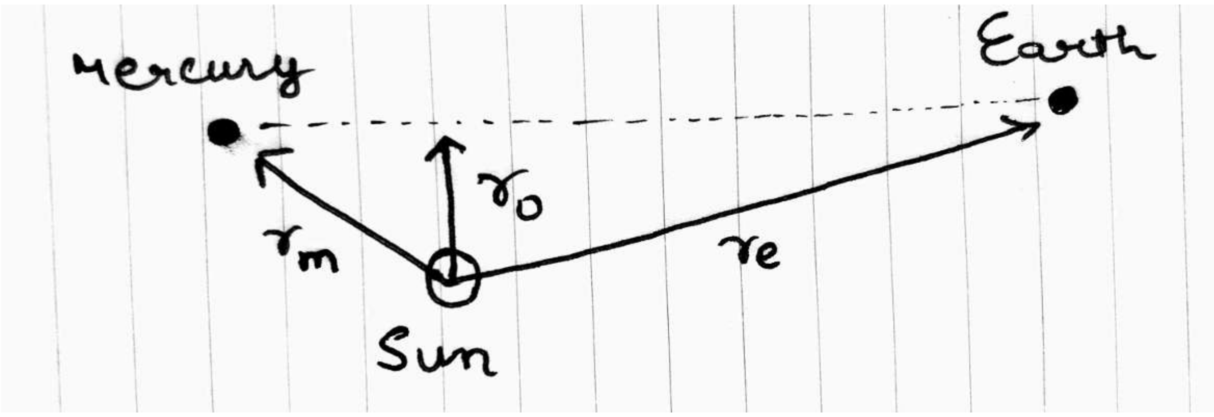
Here is a graph that explains the situation better:



Q2: Derive the Shapiro Delay Equation for a pulsar signal passing near a massive body.

A radar pulse is transmitted from Earth, narrowly bypassing the Sun and impacting the planet Mercury on its far side during a superior conjunction. The pulse then makes its way back to Earth, as illustrated schematically in the accompanying figure (not to scale).

Re denotes the distance from the Sun to Earth, Rm represents the distance from the Sun to Mercury, and Ro indicates the distance from the center of the Sun to the point of closest approach (perihelion) of the radar pulse's trajectory.



Some authors utilize Schwarzschild coordinates, while others prefer isotropic coordinates. Additionally, there are variations in how results are presented; some authors use coordinate time, while others incorporate adjustments to account for gravitational time dilation and the motion of clocks on Earth. Furthermore, a distinction exists between those who adopt a "straight path" approximation and those who employ the actual (curved) null geodesic path.

The first two differences are relatively straightforward and can be easily reconciled. However, the third distinction—between the "straight" path and the geodesic path—presents a more nuanced issue. There are numerous incorrect assertions in the literature suggesting that the "straight" path approximation does not significantly affect the results to the first order in m/r. Contrary to this belief, it does indeed produce a first-order difference, as will be elaborated below, and the oversight of this factor contributes to considerable confusion surrounding the topic.

Originally, the equation presented by Shapiro was:

$$\Delta t_r = \frac{4m}{c}\left\{\ln\left[\frac{x_p + \sqrt{x_p^2 + r_0^2}}{-x_e + \sqrt{x_e^2 + r_0^2}}\right] - \frac{1}{2}\left[\frac{x_p}{\sqrt{x_p^2 + r_0^2}} + \frac{2x_e + x_p}{\sqrt{x_e^2 + r_0^2}}\right]\right\}$$

Here m is the mass of the sun in geometrical units, xe is the distance along the line of line between the earth and the perihelion, xp as the distance between target planet Mercury and the perihelion, he noted that in the limit as x_p and x_e are much greater than r_0 the expression for the delay reduces to:

$$\Delta t_r = \frac{4m}{c}\left\{\ln\left[\frac{4x_p x_e}{r_0^2}\right] - \frac{3x_e + x_p}{2x_e}\right\}$$

The derivation flows such that:

First we will show how Shapiro's formula can be derived from the premises he stated, i.e., a "straight path" in terms of Schwarzschild coordinates. For convenience we use polar Schwarzschild coordinates, in terms of which we have the metric relation for null paths.

$$(dt)^2 = \frac{1}{\left(1-\frac{2m}{r}\right)^2}(dr)^2 + \frac{1}{1-\frac{2m}{r}} \cdot r^2 (d\theta)^2$$

$$r_0 = r\sin\theta .$$

$$\Rightarrow r\cos\theta \cdot d\theta + dr \sin\theta = 0$$

so, $$r^2 (d\theta)^2 = \tan(\theta)^2 \cdot (dr)^2 =$$

$$\frac{r_0^2}{r^2 - r_0^2} (dr)^2$$

$$dt = \sqrt{\frac{1}{\left(1-\frac{2m}{r}\right)^2} + \frac{1}{\left(1-\frac{2m}{r}\right)} \cdot \left[\frac{r_0^2}{r^2-r_0^2}\right]}\, dr$$

$$dt = \frac{r + 2m - \dfrac{r_0^2 m}{r^2} \cdot dr}{r^2 - r_0^2}$$

after using elementary integrals we get :

$$\Delta t = 2\left[\sqrt{r_e^2 - r_0^2} + \sqrt{r_p^2 - r_0^2}\right]$$

$$+ 4m\left[\ln\left[\frac{r_e + \sqrt{r_e^2 - r_0^2}}{r_0}\right] + \ln\left[\frac{r_p + \sqrt{r_p^2 - r_0^2}}{r_0}\right]\right]$$

$$- 2m\left[\frac{\sqrt{r_e^2 - r_0^2}}{r_e} + \frac{\sqrt{r_p^2 - r_0^2}}{r_p}\right]$$

Some authors utilize Schwarzschild coordinates, while others prefer isotropic coordinates. Additionally, there are variations in how results are presented; some authors use coordinate time, while others incorporate adjustments to account for gravitational time dilation and the motion of clocks on Earth. Furthermore, a distinction exists between those who adopt a "straight path" approximation and those who employ the actual (curved) null geodesic path.

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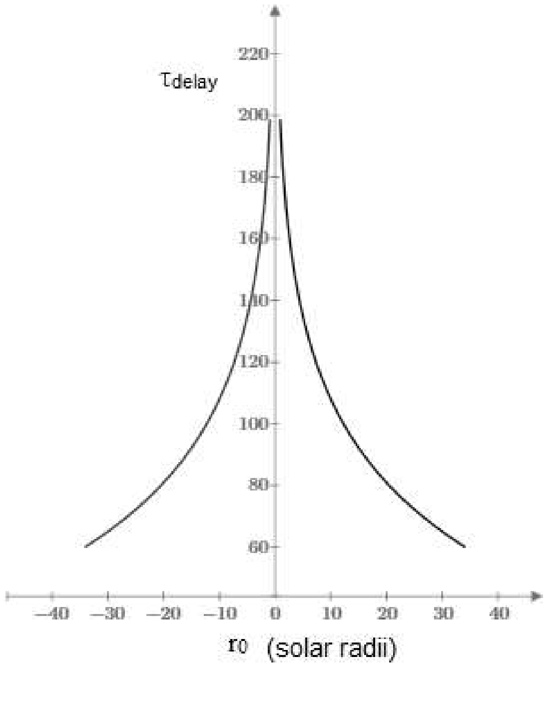
$$\Delta t_{delay} = \frac{4m}{c}\left[\ln\left(\frac{4r_e r_p}{r_0^{\ 2}}\right) - 1\right]$$

This graph explains the situation better:



Q3: If a hypothetical exoplanet at 0.1 AU causes a delay in pulse arrival, calculate the magnitude of the effect.

MON TUE WED THU FRI SAT SUN DATE / /

Assuming an Earth mass exoplanet:

$$\Delta t_{delay} = \frac{a_p}{c}$$

where 'a_p' is the semi major axis of the pulsar's rotation around the system's centre of mass and 'c' is the speed of light.

$$a_p = \frac{M_{exoplanet}}{M_{pulsar}} \times a_{exoplanet} \quad 0.1\,AU$$

$$= \frac{3 \times 10^{-6} \cancel{M_\odot}}{1.4 \cancel{M_\odot}} \times 0.1$$

$$a_p \Rightarrow 2.14 \times 10^{-7}, \quad \boxed{\Delta t = 1.1 \times 10^{-4}\ secs}$$

We can say that T delay = 1.1 x 10^{-4} seconds

Chapter 3: Relativity in Action: Time and Jets

As they tuned their devices, the pulsar released an enormous jet of energy, with a speed that approached light speed. Nobita gazed at it in awe. "That jet is seriously fast! Is time even flowing like this outside?"

"Not in the least," Doraemon said, flicking on his Relativity Goggles. "Both the gravitational field of the pulsar and the speed of the jet bend time. Let's calculate precisely how much.

Q1: Determine the dark matter density distribution around a pulsar using the G2=Gravitational rotation curve theory.

Galaxy Rotation Curves

Much of the evidence comes from the motions of galaxies. A galaxy rotation curve is a plot of the orbital velocities (i.e., the speeds) of visible stars or gas in that galaxy versus their radial distance from that galaxy's center.
https://www.sciencedirect.com/topics/physics-and-astronomy/galaxy-rotation-curves

Dark Matter Distribution

Dark Matter Distribution refers to how dark matter is spread out in our local neighborhood, including the dark matter halo of our galaxy, the Local Group, and the broader Local Universe. It involves studying the abundance of substructures, void regions, and low rotational velocity galaxies through numerical simulations and observational data to understand the physical properties of dark matter particles.
https://www.sciencedirect.com/topics/physics-and-astronomy/dark-matter-distribution

Now let us try and determine the dark matter energy distribution around a pulsar using the gravitational rotational curve theory.

The procedure sits as follows:
First you need some sort of model (the dependence of density on r) for how the dark matter behaves (e.g., a Navarro, Frenk & White profile) with some free parameters like a scale radius and central density and then some observational constraints on the normal matter density profile, perhaps from how much light it emits. The sum of these two will be the total gravitating matter density.

To begin, it is necessary to address the equations of motion by considering the gravitational forces acting on an object at radius r. This analysis will enable the prediction of the circular orbital speed at that radius. Subsequently, this model, in conjunction with the observed rotation curves, is incorporated into a framework such as a **Markov Chain Monte Carlo (MCMC) Bayesian estimator.** This process yields the posterior probability distribution for the free parameters within your dark matter model.

This overview serves as a general guideline; the precise details will vary based on the specific data available.

An alternative, albeit simplified approach, which relies on a significant approximation, also requires an understanding of the amount of normal matter present within a given radius r. From this point, one can assume spherical symmetry, allowing for further analysis.

$$\frac{GM(<r)}{r^2} = \frac{v(r)^2}{r}$$

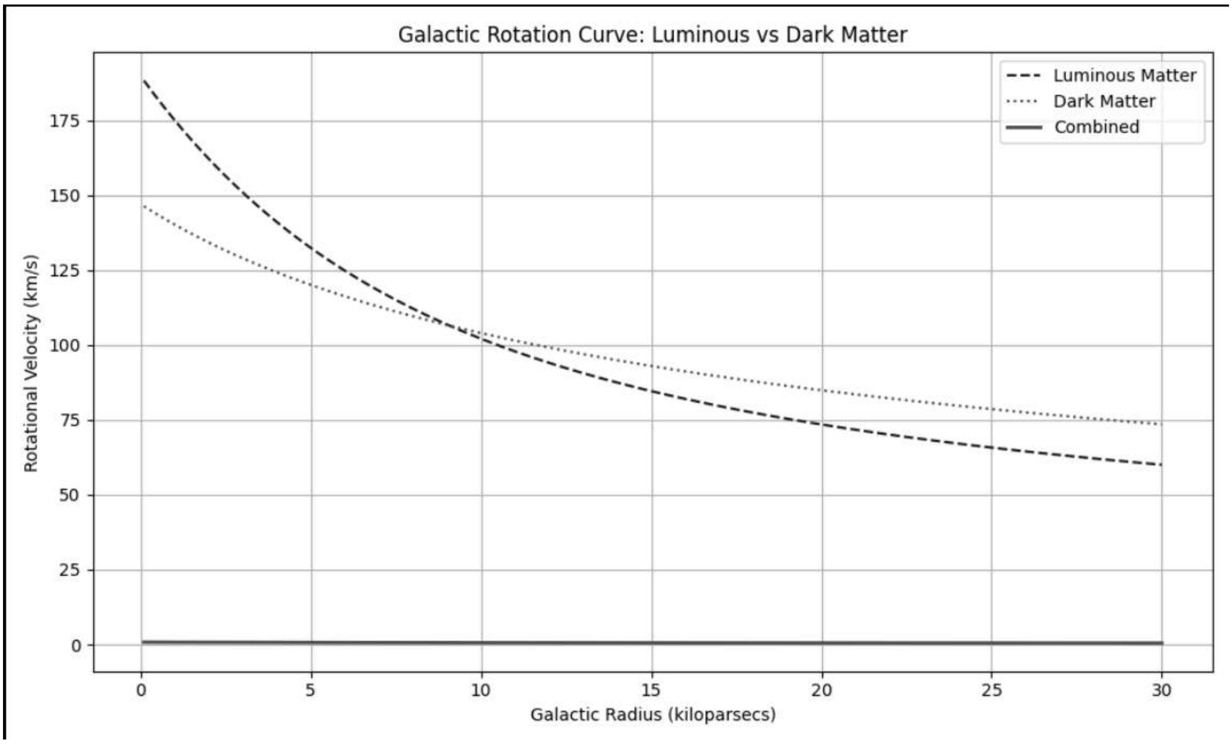
. From each discrete value you have of $v(r)$ you can get an estimate of $M(<r)$, the mass inside radius r. If you subtract off your estimate for how much normal matter there is within that radius you get an estimate of how much dark matter there is within r. Differentiating $M_{\text{dark}}(<r)$ with respect to r and dividing by $4\pi r^2$ gives you an approximation for the dark matter density profile.

https://www.researchgate.net/publication/261475251_The_Local_Dark_Matter_Density

2. In what visible ways would a pulsar travelling through a dense region of dark matter appear to display any irregularity?

A python program link lies here: **Python Program Link**

SOURCES:https://arxiv.org/abs/1904.05954?utm_source
https://www.mdpi.com/1099-4300/27/2/194?utm_source
https://journals.aps.org/prd/abstract/10.1103/PhysRevD.105.104020?utm_source



The result, in the form of a graph

Q3: In what visible ways would a pulsar travelling through a dense region of dark matter appear to display any irregularity?

If a pulsar were traveling through a dense region of dark matter, its observable properties might exhibit certain irregularities due to gravitational and exotic interactions. Some key visible effects could include:

1. Irregularities in Pulse Timing (Shapiro Delay & Doppler Shifts)

The gravitational effects of dark matter may influence the motion of pulsars, resulting in fluctuations in their spin-down rates and producing irregularities in the timing of pulse arrivals. The phenomenon known as Shapiro delay, which refers to gravitational time delay, could extend the time it takes for pulses to arrive due to the enhanced curvature of spacetime. Furthermore, if dark matter exists in clumps, the pulsar's trajectory through these regions could lead to minor yet measurable variations in its orbital characteristics, resulting in discrepancies from the anticipated periodic behavior.

2. Unexplained Changes in Spin-Down Rate

A pulsar navigating through an area rich in dark matter may encounter additional energy dissipation due to potential interactions with axions or other theoretical dark matter entities. This interaction could lead to an unexpected rise in the spin-down rate (\dot{P}), diverging from conventional dipole radiation predictions.

3. Lensing Effects on Pulses

In regions of high dark matter concentration, the associated gravitational field may produce gravitational lensing effects, resulting in the magnification or distortion of pulse signals. Consequently, the pulses could appear more intense or experience temporary delays, contingent upon the arrangement of dark matter along the observational path.

4. Increased Scattering and Dispersion Effects

Should dark matter exhibit weak interactions with standard model particles, it may generate plasma-like phenomena or interact with nearby interstellar matter, resulting in unforeseen pulse dispersion or scattering. This effect could resemble variations in electron density within the interstellar medium.

Chapter 4: Stretching Horizons

Chapter 4: Stretching Horizons

The Door to Anywhere opened to a great universe. "Dark energy!" said Doraemon. "It stretches the cosmos and pulsars tell it.

"How can a pulsar do this?" Nobita asked, looking puzzled.

"Cosmic clocks," Doraemon said. "Their timing data show the rate of cosmic expansion!".

1. Derive the Lorentz factor for a pulsar jet with a speed of 0.9c

The Lorentz factor describes realistic effects such as time dilation and length contractions for objects moving at significant portions of the speed of light(c). For a pulsar jet moving at v = 0.9c, the Lorentz factor is derived as follows : -

$$\cancel{\gamma =} \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Substitute, $v = 0.9c$,

$$\gamma = \frac{1}{\sqrt{1-\frac{(0.9c)^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{0.19}}, \qquad \sqrt{0.19} \approx 0.4359$$

$$\boxed{\gamma = \frac{1}{0.4359} \approx 2.294}$$

This implies that time in the jet would dilate by this factor for the observer , for example 1 second in jet would mean 2.29 seconds for the observer.
https://en.wikipedia.org/wiki/Lorentz_factor
https://resources.wolframcloud.com/FormulaRepository/resources/Lorentz-Factor

2. Give an account of the phenomenon of apparent superluminal motion in relativistic jets

Apparent superluminal motion in relativistic jets is an astronomical phenomenon , which is also an observational illusion where relativistic jets , high speed streams of ionized matter ejected from astrophysical objects like black holes , neutron stars etc, appear to move faster than the speed of light when observed from earth , even though the actual jet speed is below the speed of light. It arises from a combination of realitivistic effects rather then actual superluminal travel.

Mathematical Basis

Relativistic Speeds :- Jets are ejected at velocities approaching the speed of light. At such speeds time dilation and length contraction become significant.

Viewing Angle :- If jet is ejected at small angle with respect to the observers line of sight say theta , light emitted from approaching regions of the jet will reach Earth sooner than receding regions. This compresses the observed time between emissions , creating the illusion of superlimuinal motion.

Critical Angle :- The maximum apparent velocity occurs near Theta tends to 1 / Lorentz factor.

https://www.sciencedirect.com/topics/physics-and-astronomy/apparent-superluminal-motion#:~:text=The%20evidence%20for%20superluminal%20motion,is%20constant%20along%20the%20jet.
https://en.wikipedia.org/wiki/Superluminal_motion

Apparent Velocity Formula :- The Apparent Transverse velocity is

$$\text{apparent speed} = \frac{\text{distance}}{\text{interval}}$$

$$= \frac{v(1\ \text{yr})\sin(\theta)}{(1\ \text{yr})\,[1-(v/c)\cos(\theta)]}$$

$$= \frac{v\sin(\theta)}{[1-(v/c)\cos(\theta)]}$$

Observational Evidence :-
Quasars: The jet in quasar 3C 273 was one of the first observed cases.
AGN Jets: The blazar 3C 279 exhibits superluminal motion in radio observations (Jorstad et al., 2017).
Galactic Sources: The microquasar SS 433 and the M87 black hole jet (observed by the Event Horizon Telescope) show superluminal features.

Key Implications
Relativity Compliance: No physical object or information exceeds the effect is purely geometric.
Probing Jet Physics: Observations constrain jet velocities, angles, and energy mechanisms (e.g., magnetic fields, accretion disk dynamics).

3. Compare gravitational time dilation between the pulsar surface and an observer in the jet.

Gravitational time dilation, predicted by Einstein's general relativity, causes clocks to tick slower in stronger gravitational fields. Here, we compare the time experienced by an observer on a pulsar's surface to that of an observer in the jet.

1. Gravitational Time Dilation at the Pulsar Surface:-
For a neutron star (pulsar), the gravitational time dilation is governed by the Schwarzschild metric:

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{2GM}{Rc^2}}$$

$d\tau$ → Proper time (pulsar surface)
dt → Coordinate time (distant observer)

$M = 1.4 M_{\odot}$, $R \approx 10\,km$ (Pulsar parameters)

$$\frac{2GM}{Rc^2} \approx 0.413, \quad \frac{d\tau}{dt} = \sqrt{1 - 0.413} \approx 0.766$$

Interpretation - Clocks on the pulsar surface tick at approx 76.6% the rate of distant observers clock.

2. Gravitational Time Dilation for an Observer in the Jet:-

Interpretation: The jet observer experiences negligible gravitational time dilation compared to the pulsar surface.

$$\frac{d\tau_{jet}}{dt} \approx \sqrt{1 - \frac{2Gm}{Rc^2}}$$

Since, $r \gg R$, $\frac{2Gm}{rc^2} \to 0$,

$$\frac{d\tau_{jet}}{dt} \approx 1.$$

3. Direct Comparison:- The ratio of time dilation between the pulsar surface and jet observer is:

$$\frac{d\tau_{surface}}{d\tau_{jet}} \approx \sqrt{1 - \frac{2Gm}{Rc^2}} \approx 0.766$$

Conclusion: Time on the pulsar surface flows ~23.4% slower than in the jet.

Key Implications
- Pulsar Observations: Surface emission (e.g., X-ray pulsations) appears redshifted to distant observers.
- Jet Dynamics: Relativistic motion in jets introduces additional special relativistic time dilation, but gravitational effects dominate near the surface.

Sources -
https://press.princeton.edu/books/hardcover/9780691177793/gravitation
https://onlinelibrary.wiley.com/doi/book/10.1002/9783527617661
https://onlinelibrary.wiley.com/doi/book/10.1002/9783527617661

Chapter 5: The Dark Matter Dance

Their last destination was the boundary of known physics-a frontier where spacetime itself started breaking apart. "The Planck scale marks the edge of our knowledge," Doraemon said. "At these energies, pulsars could possibly give a preview into quantum gravity."

Q1: Derive the formula for Planck energy and compute its value.

① COMPTON WAVELENGTH

$$\lambda_C = \frac{h}{\frac{E}{c^2} \times c} = \frac{hc}{E}$$

② SCHWARZSCHILD RADIUS

Radius of Black hole of mass m

$$R_S = \frac{2GM}{c^2}$$

as $\boxed{m = \frac{E}{c^2}}$

$$R_S = \frac{2GE}{c^4}$$

set $\lambda_C = R_S$

$$\boxed{\frac{hc}{E} = \frac{2GE}{c^4}}$$

$$E^2 = \frac{hc^5}{2G}$$

$$\boxed{E_p = \sqrt{\frac{hc^5}{G}}}$$

Factor of 2 can be ignored as it arises from schwarzschild radius, but Planck's energy is ~~not~~ scale, not exact value.

$$E_p = \sqrt{\frac{hc^5}{G}}$$

Substituting $h = 6.626 \times 10^{-34}$ J/s
$c = 3.0 \times 10^{8}$ m/s
$G = 6.674 \times 10^{-11}$

$$\boxed{E_p \simeq 4.91 \times 10^{9}\ J}$$

planck's energy

Q2: What would the Schwarzschild radius for the black hole formed from a Planck-mass object be?

The Schwarzschild radius is given by

$$R_S = \frac{2GM}{c^2}$$

for Planck-mass object $(M = m_p)$, Substitute the Planck mass formula

$$m_p = \sqrt{\frac{\hbar c}{G}}$$

$$R_S = \frac{2G}{c^2}\sqrt{\frac{\hbar c}{G}} = 2\sqrt{\frac{\hbar G}{c^3}}$$

~~Doing~~ After calculation

$$\simeq 8.09 \times 10^{-35}\ m$$

:

SOURCES:https://vixra.org/pdf/1512.0496v1.pdf?utm_source

Q3:Write a Python program that plots a curve describing the galaxy's rotation curve, including observed and predicted velocities curves.

Python Program Link-3

SOURCES:https://ircamera.as.arizona.edu/NatSci102/NatSci/lectures/darkmatter.htm?